HW 3

Problem 2.5

from 2.2, $R' = 0.78784$
$L' = 1.3863 \times 10^{-7}$
$G' = 9.0647 \times 10^{-3}$
$C' = 361 \times 10^{-12}$
$f = 10^9$

$u_p = f\lambda = \frac{\omega}{\beta}$

$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$

Ref: Eq. 2.22, 2.24, 2.25, 9.6

$\gamma = \alpha + j\beta \quad \gamma^2 = (R + j\omega L)(G' + j\omega C')$

$\alpha = Re[\sqrt{\gamma^2}], \quad \beta = Im[\sqrt{\gamma^2}]$

$\omega = 2\pi \times 10^9$

$R' + j\omega L' = 0.78784 + j871.04 = 871 \angle 89.95^\circ$

$G' + j\omega C' = 0.0090647 + j2.27 = 2.268 \angle 89.77^\circ$

$\gamma^2 = 1975.4 \angle 179.7^\circ$

$\gamma = 44.45 \angle 89.86$

$\gamma = 0.109 + j44.5$

$\rightarrow \alpha = 0.109 \frac{Np}{m}$

$, \beta = 44.5 \frac{rad}{m}$

$u_p = \frac{\omega}{\beta} = \frac{6.28 \times 10^9}{44.5}$

$u_p = 1.41 \times 10^8 \frac{m}{s}$

$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{0.79 + j872.92}{9.1 \times 10^{-3} + j2.267}} = 19.6 + j0.03 \ \Omega$

2.6

Distortionless line condition: $R'C' = L'G'$

$$Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} \rightarrow Z_0 = \sqrt{\frac{(R'C'+j\omega L'C')}{C'} \frac{L'}{(\underbrace{G'L'}_{=R'C'}+j\omega L'C')}}$$

$$\Rightarrow Z_0 = \sqrt{\frac{L'}{C'}}$$

$$\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')} \Rightarrow \gamma = \sqrt{\left(\frac{R'C'+j\omega L'C'}{C'}\right)\left(\frac{\underbrace{G'L'}_{=R'C'}+j\omega L'C'}{L'}\right)}$$

$$\Rightarrow \gamma = \sqrt{\frac{(R'C'+j\omega L'C')^2}{L'C'}} = \frac{R'C'+j\omega L'C'}{\sqrt{L'C'}}$$

$$\gamma = \underbrace{\frac{R'C'}{\sqrt{L'C'}}}_{\alpha} + j\underbrace{\frac{\omega L'C'}{\sqrt{L'C'}}}_{\beta} = \frac{L'G'}{\sqrt{L'C'}} + j\omega\sqrt{L'C'}$$

$$\rightarrow \alpha = \frac{L'G'}{\sqrt{L'C'}} = G'\sqrt{\frac{L'}{C'}} = R'\sqrt{\frac{C'}{L'}} = R'\sqrt{\frac{G'}{R'}} = \sqrt{R'G'}$$

$$\beta = \omega\sqrt{L'C'}$$

Problem 2.7

$Z_0 = 50$, $\alpha = 20\times10^{-3}$, $u_p = 2.5\times10^{8}\ \mathrm{m/s}$, $f = 100\,\mathrm{MHz}$

$R'C' = L'G'$ $\quad u_p = f\lambda = \frac{\omega}{\beta} \rightarrow \beta = \frac{\omega}{u_p}$

$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}$ $\quad \beta = \omega\sqrt{L'C'} = \omega L'\sqrt{\frac{C'}{L'}} = \frac{\omega L'}{Z_0}$

$20\times10^{-3} = R'\frac{1}{Z_0} \rightarrow R' = 1\ \frac{\Omega}{m}$

$L' = Z_0\frac{\beta}{\omega} = \frac{Z_0}{u_p} = \frac{50}{2.5\times10^8} = 20\times10^{-8} = 200\ \frac{nH}{m}$

$C' = \frac{L'}{Z_0^2} = \frac{2\times10^{-7}}{2500} = 80\ \frac{pF}{m}$

$G' = \frac{\alpha^2}{R'} = \frac{400\times10^{-6}}{1} = 400\times10^{-6} = 400\,\mu S/m$

$\lambda = \frac{u_p}{F} = \frac{2.5\times10^8}{10^8} = 2.5\,m$

Problem 2.9

$Z_0 = 40\Omega$

$\alpha = 0.02$ Np/m

$\beta = 0.75$ rad/m $\Rightarrow \gamma = \alpha + j\beta = 0.02 + j0.75$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} \rightarrow R' + j\omega L' = \gamma Z_0 = 40(0.02 + j0.75)$$

$$= 0.8 + j30$$

$\rightarrow R' = 0.8 \ \Omega/m$

$\omega L' = 30 \qquad \omega = 2\pi f = 2.5 \times 10^8 \pi$

$L' = \frac{30}{2.5\pi} \times 10^{-8} = 38.2 \text{ nH/m}$

Eq. 2.22, p 44

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\gamma = \sqrt{(\gamma Z_0)(G' + j\omega C')} \rightarrow \gamma^2 = \gamma Z_0 (G' + j\omega C')$$

$$G' + j\omega C' = \frac{\gamma}{Z_0} = \frac{\alpha + j\beta}{Z_0}$$

$$= \frac{0.02 + j0.75}{40}$$

$$= 0.5 \times 10^{-3} + j\frac{75}{4} \times 10^{-3}$$

$G' = 0.5 \text{ mS/m}$

$\omega C' = \frac{75}{4} \times 10^{-3}$

$C' = \frac{75}{10\pi} \times 10^{-11} = 23.9 \text{ pF/m}$

2.11

$\epsilon_r = 2.25$, $a = 1.2 \times 10^{-3}$ m (inner conductor radius)

a) $Z_0 = 50\,\Omega$

lossless $\rightarrow$ $Z_0 = \sqrt{\frac{L'}{C'}}$ $\quad L' = \frac{\mu}{2\pi} \ln \frac{b}{a}$

$C' = \frac{2\pi\varepsilon}{\ln(\frac{b}{a})}$

$\rightarrow Z_0^2 = \frac{\mu}{\varepsilon}\left[\frac{\ln(\frac{b}{a})}{2\pi}\right]^2$

assume $\mu = \mu_0$ $\rightarrow$ $\ln(\frac{b}{a}) = 2\pi\sqrt{\frac{\varepsilon_r \varepsilon_0}{\mu_0}}\ Z_0$

$\ln(\frac{b}{a}) = 2\pi\left(\frac{2.25 \times 8.854 \times 10^{-12}}{4\pi \times 10^{-7}}\right)^{\frac{1}{2}}(50)$

$\rightarrow \ln(\frac{b}{a}) = 1.25$

$\frac{b}{a} = e^{1.25} \rightarrow b = 1.2\, e^{1.25}$ mm

$b = 4.19$ mm

b) $u_p = \frac{1}{\sqrt{\mu\varepsilon}}$ $\quad$ Eq. 2.40, p53

$u_p = \frac{1}{\sqrt{2.25 \times 8.854 \times 10^{-12} \times 4\pi \times 10^{-7}}} = \frac{1}{\sqrt{250.3 \times 10^{-19}}} = 2 \times 10^{8}\ \frac{m}{s}$

Problem 2.12

lossless line, $Z_0 = 50\,\Omega$

$Z_L = 30 - j50\,\Omega \rightarrow$ Series RC load $\quad R = 30\,\Omega$

$\frac{1}{\omega C} = 50\,\Omega$

$\lambda = 8\text{ cm} = 0.08\text{ m}$

a) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j50 - 50}{30 - j50 + 50} = \frac{-20 - j50}{80 - j50} = -\left(\frac{2 + j5}{8 - j5}\right)$

$\Gamma = 0.571 \underline{\angle -79.8^\circ}$

b) -

$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1.571}{0.429} = 3.66$

c) z for V_{max} nearest the load

$\theta_r = -79.8^\circ$ is negative nearest load is n=1

$-z = l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} = \left(\frac{-1.39}{2\pi} + 1\right)\left(\frac{\lambda}{2}\right)$

$= 0.778\left(\frac{8}{2}\right) = -3.11$ cm

d) z for I_{max} nearest the load

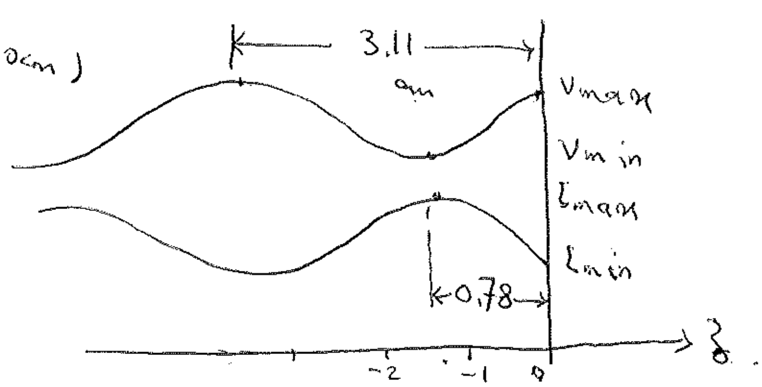
I_{max} is where V_{min} occurs or $\frac{\lambda}{4}$ on either Side (unless $l_{max} < \frac{\lambda}{4}$)

$l_{max} = 3.11\text{cm} > \frac{\lambda}{4}$ $\left(\frac{\lambda}{4} = 2.0\text{cm}\right)$

Eq 2.58, p.60 $\quad l_{min} = l_{max} - \frac{\lambda}{4}$

$l_{min} = 3.11 - 2 = 1.11$ cm

$z_{min} = -1.11$ cm



2.13

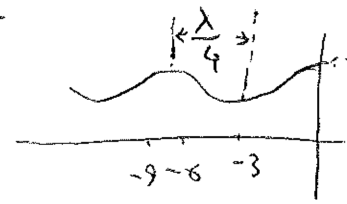
$Z_0 = 150\ \Omega$

$S = 3$ min to max $= \frac{\lambda}{4} = 6$ cm $\rightarrow \lambda = 24$ cm

lossless

$Z_L = ?$

$S = \frac{1+|\Gamma|}{1-|\Gamma|} \rightarrow |\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{1}{2}$



$|\Gamma| = \frac{1}{2}$

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow Z_L = \frac{1+\Gamma}{1-\Gamma} Z_0$

$-z = l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} \rightarrow \theta_r = 4\pi \frac{l_{max}}{\lambda} - 2\pi n$

if $\theta_r \geq 0$, $n = 0$ is a possibility

$\theta_r = 4\pi\left(\frac{9}{24}\right) - 0 = 4.71$ rad $= 270^\circ \geq 0$

$\therefore$ $n = 0$ works (also agrees with sketch above!)

$\Gamma = |\Gamma| \angle \theta_r \rightarrow \Gamma = \frac{1}{2} \angle 270^\circ = \frac{1}{2} \angle -90^\circ = 0 - j\frac{1}{2}$

$Z_L = \frac{1 - j\frac{1}{2}}{1 - (-j\frac{1}{2})}(150) = \frac{2 - j1}{2 + j1}(150) = \frac{(2-j1)(2-j1)}{5}(150)$

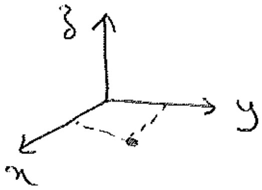
$= \frac{3 - j4}{5}(150)$

$Z_L = 90 - j120\ \Omega$

3.19

a) $P_1(1,2,0)$ Cylindrical, z is 0 $\quad r=\sqrt{1^2+2^2}=\sqrt{5}$

$\phi = tg^{-1}\left(\frac{2}{1}\right) = 63.43$



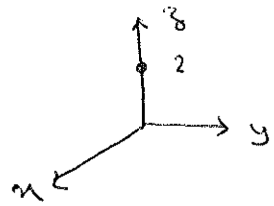
cyl. $P_1(\sqrt{5}, 63.4, 0)$

spherical: $R = r$, $\theta = 90$, ϕ is the same

sph. $P_1(\sqrt{5}, 90^\circ, 63.4^\circ)$

b) $P_2(0,0,2)$

Cylindrical: z is still 2, $r=0$, $\phi=0$

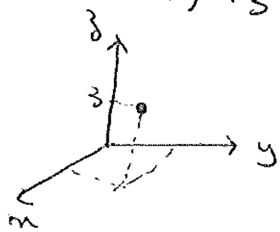


cyl. $P_2(0,0\,2)$

Sph. $R=2$, $\theta=0^\circ$, $\phi=0^\circ$

sph. $P_2(2, 0^\circ, 0^\circ)$

c) $P_3(1,1,3)$



cyl. z is still 3 $\quad r=\sqrt{1^2+1^2}=\sqrt{2}$

$\phi = tg^{-1}\left(\frac{1}{1}\right) = 45$

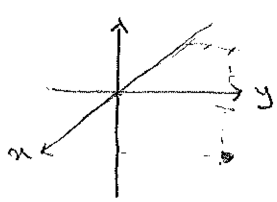
cyl. $P_3(\sqrt{2}, 45^\circ, 3)$

spherical: $R=\sqrt{1^2+1^2+3^2}=\sqrt{11}$, ϕ is still 45°

$R\cos\theta = 3 \rightarrow \theta = \cos^{-1}\left(\frac{3}{\sqrt{11}}\right) = 25.24$

sph. $P_3(\sqrt{11}, 25.24^\circ, 45^\circ)$

d) $P_4(-2,2,-2)$



Cyl. z is still -2, $r=\sqrt{2^2+2^2}=2\sqrt{2}$

$\phi = 135^\circ$

Cyl $P_4(2\sqrt{2}, 135^\circ, -2)$

spherical: ϕ is still 135, $R=\sqrt{2^2+2^2+2^2}=2\sqrt{3}$

$R\cos\theta = z = -2 \qquad \theta = \cos^{-1}\left(\frac{-2}{2\sqrt{3}}\right)$

sph. $P_4(2\sqrt{3}, 125.3, 135)$

Problem 3.21

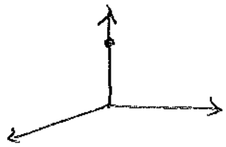
a) $P_1(5, 0°, 0°)$ $R=5,\ \theta=0°,\ \phi=0° \rightarrow z=5,\ x=y=0$

or $r=0,\ \phi=0$

cyl $P_1(0, 0, 5)$

b) $P_2\ (5, 0°, \pi)$ $R=5,\ \theta=0°,\ \phi=180° \rightarrow z=5,\ x=y=0$

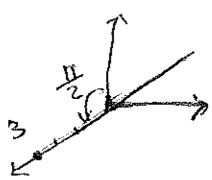
$r=0,\ \phi=\pi$



cyl $P_2\ (0, \pi, 5)$

c) $P_3\ (3, \frac{\pi}{2}, 0)$ $R=3,\ \theta=90°,\ \phi=0° \rightarrow z=0,\ x=-3,\ y=0$

$r=3,\ \phi=0°$



cyl $P_3\ (3, 0, 0)$